Sept. 1996 hep-ph/9609286 FT–UAM 96/40

# Radiatively induced light right-handed stop

**Alessandro Strumia**

*Departamento de Física Teórica, Universidad Autónoma de Madrid, 28049, Madrid, España* and *INFN, sezione di Pisa, I-56126 Pisa, Italia*



## Abstract

A right-handed stop not much heavier or even lighter than the $Z$ boson has today desirable phenomenological consequences. We study how it can result within the usual radiative scenario of electroweak symmetry breaking. A restriction on the gaugino mass parameters, $M_2 \lesssim 0.3 m_{10}$, arises if soft terms satisfy relations suggested by unification theories. Moreover, requiring to get a light stop without unnatural fine-tunings below the per-cent level, we obtain another more interesting upper bound on the chargino mass, $M_\chi \lesssim M_Z$, and we derive interesting conclusions about the masses of the gluino, $M_3 \sim (150 \div 300)\,\mathrm{GeV}$, of the heavy stop, $M_{\tilde{T}} \sim (250 \div 500)\,\mathrm{GeV}$, and of the left/right mixing angle in the stop sector, $|\theta_{\tilde{t}}| \lesssim 0.3$.

**1** There are today independent phenomenological indications that suggest the presence of a light, mostly right-handed, stop state with mass around the $Z$-pole.

To begin, it has recently pointed out in [1], that the electroweak phase transition can be sufficiently strongly first order (so that the observed baryon asymmetry can be generated during the electroweak phase transition) for acceptable values of the higgs mass if a stop state is light, or, more exactly, if its soft mass term is sufficiently small, $|m^2_{\tilde{t}_R}| \sim M_Z^2$.

Moreover, a mostly right-handed stop with mass $M_{\tilde{t}} \lesssim 100\,\mathrm{GeV}$, together with a similarly light chargino, mediates supersymmetric corrections to the electroweak precision observables that explain the discrepancy between the measured value of the $Z \to b\bar{b}$ width and its Standard Model (SM) prediction without affecting the other (successful) SM-predictions [2]. The consequent decrease of the predicted hadronic $Z$ width would reduce the extracted value of the strong coupling constant, $\alpha_3(M_Z)$, as suggested by its low energy determinations [2, 3].

In both cases these desired features are specific of the small $\tan\beta$ region. Even if a definitive numerical calculation is still lacking, with a moderate amount of mixing between the $\tilde{t}_L$ and $\tilde{t}_R$ states, the lightest stop $\tilde{t}$ can probably be sufficiently light and right-handed so that to solve both problems at the same time without requiring a too large negative value of the stop soft term $m^2_{\tilde{t}_R} \gtrsim -m_t^2$ that would give rise to charge and color breaking minima of dangerous kind [4].

**2** It is well known that since the higgs doublet that gives mass to up-quarks, $h^{\mathrm{u}}$, is the smallest multiplet involved in the Yukawa coupling of the top quark, its soft mass term, $m^2_{h^{\mathrm{u}}}$, is easily driven to be negative by renormalization effects. For universal soft terms at the unification scale this happens in all the parameter space. We will study in which regions of the parameter space this happens also for the soft mass$^2$ term, $m^2_{\tilde{t}_R}$, of the second smallest multiplet involved in the top-quark Yukawa coupling, the right-handed stop. We will see that requiring to get a light stop in the ordinary successful scenario for the radiative breaking of the electroweak gauge symmetry, implies, among the other things, the presence of a very light chargino in the spectrum and gives strongly favored values for the gluino mass, for the heavy stop mass and for the left/right mixing angle in the stop sector.

The most important new feature regards naturalness bounds [5]: since we now want to fix *two* different combinations of soft terms to be around at the $Z$ scale, the presence of heavy supersymmetric parti-

cles (around at 1 TeV) in the spectrum requires now strongly unnatural fine-tuning of the various relevant parameters.

We only assume that the gaugino masses satisfy GUT-like relations and that the soft terms are point-like operators up to the unification scale, as happens, for example, if they are mediated by supergravity couplings [6]*. In this case, near the infrared fixed point of the top quark Yukawa coupling and for moderate values of $\tan\beta$, the $Z$-boson mass is obtained, from the MSSM minimization conditions, as a sum of different terms

$$M_Z^2 \approx -2\mu^2 + \mathcal{O}(1) m_0^2 + \mathcal{O}(10) M_2^2 \qquad (1)$$

where $M_2$ is the $\mathrm{SU}(2)_L$ gaugino mass parameter and $\mu$ is the '$\mu$ term', both renormalized at the weak scale, while $m_0^2$ is a typical scalar mass squared at the unification scale. We will consider unnatural a situation where the single contributions to $M_Z^2$ in eq. (1) are much bigger than their sum, and define the inverse 'fine tuning' $1/f' = \Delta'$ as the largest single contribution to $M_Z^2$ in units of $M_Z^2$. In ordinary situations, and near the infrared fixed-point of the top quark Yukawa coupling, this simplified definition does not differ from other reasonable and more sophisticated choices [5] that consider the stability of $M_Z^2$ with respect to variations of a set of parameters chosen as the 'fundamental' ones. For this reason we will continue to employ the term 'fine tuning' in our discussion of the naturalness bounds. In particular, since the right handed term of eq. (1) contains a $\mathcal{O}(10) M_2^2$ term, we have $\Delta' \gtrsim 10 M_2^2/M_Z^2$, so that, imposing the naturalness condition $\Delta' < \Delta_{\rm lim}$, implies

$$M_2 \lesssim 1\,\mathrm{TeV}\,(\Delta_{\rm lim}/10^3)^{1/2}.$$

Now we also want to obtain a light right-handed stop. Since $m_{\tilde{t}_R}^2$ is obtained as

$$m_{\tilde{t}_R}^2 \approx -\mathcal{O}(0.3) m_0^2 + \mathcal{O}(6) M_2^2 \qquad (2)$$

we need, in particular, another $\Delta'' \gtrsim 6 M_2^2/M_Z^2$ inverse fine-tuning.

For this reason, imposing a naturalness bound on the *total* 'fine tuning', $\Delta = \Delta'\Delta'' < \Delta_{\rm lim}$, gives rise to an upper bound on the sparticle masses that is stronger and less dependent on the personal choice of the maximum tolerated 'fine tuning', $1/\Delta_{\rm lim}$. In particular a significant bound on $M_2$ is obtained:

$$M_2 \lesssim 2 M_Z\,(\Delta_{\rm lim}/10^3)^{1/4}.$$

This means that *a very light chargino is consequence of requiring a light stop*. More generally, interesting

*We will see that in the opposite case [7] it is difficult to obtain a light right-handed stop.

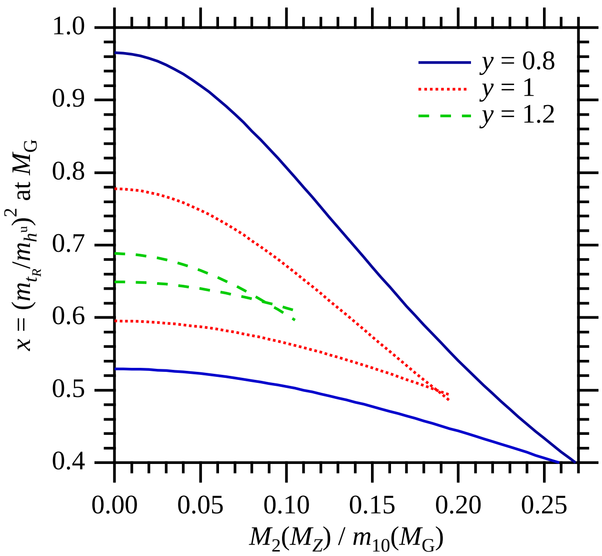


Figure 1: *A negative value of the stop soft term $m_{\tilde{t}_R}^2$ at the Fermi scale is obtained in the region inside the lines, plotted for different values of $y \equiv m_{\tilde{t}_R}^2/m_{\tilde{Q}_3}^2$ at $M_{\rm GUT}$, $A_t = 0$ at $M_{\rm GUT}$, and near the infrared fixed point for the top Yukawa coupling.*

naturalness upper bounds can be derived on the parameters, like $M_2$ and $m_0$, that appear in both the expression for $M_Z^2$ and for $m_{\tilde{t}_R}^2$. On the contrary the $\mu$-term — and consequently the masses of charged and pseudoscalar Higgs fields – are less strongly constrained†. A numerical analysis is of course necessary to clarify the strength of these double 'fine-tuning' bounds.

**3** Let us now consider these issues in a more quantitative way. We parameterize the relevant boundary condition at the unification scale as

$$m_{10}^2 \equiv m_{\tilde{t}_R}^2 = y m_{\tilde{Q}_3}^2 = x m_{h^{\rm u}}^2 \qquad (3)$$

where $x$ and $y$ are unknown numbers. Even if we will keep them arbitrary, we reasonably expect that $y \approx 1$ in a unified model. In fact, unless the chiral families mix with extra vector-like states at the unification scale in such a way that $\tilde{Q}_3$ is not unified with $\tilde{t}_R$, the equality between their GUT-scale soft masses, $m_{\tilde{Q}_3}$ and $m_{\tilde{t}_R}$, can only be broken by (small) GUT threshold corrections. On the contrary there is no reason for imposing $x = 1$, because, even if were justifiable at the Planck scale, this choice would be spoiled by (unknown) GUT renormalization effects

† Here we are considering the $\mu$ term as an independent parameter, even if we expect that in a natural theory $\mu$ vanishes in the supersymmetric limit. In unified models with this property, $\mu$ can naturally be obtained as a model-dependent linear combination of the dimension-one soft terms [8].

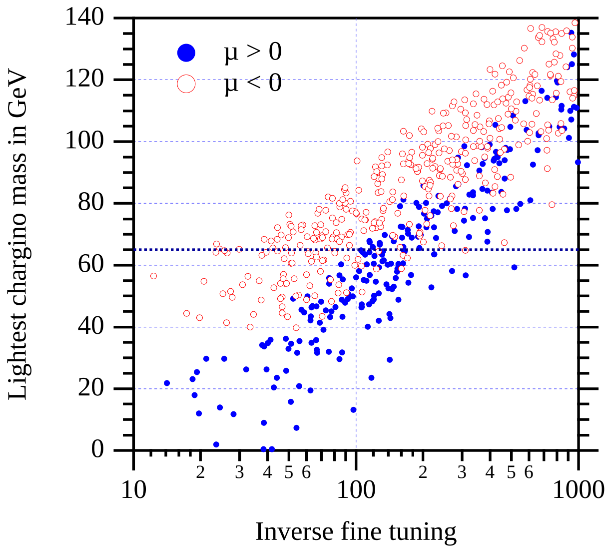


Figure 2: *Fine tuning as function of the chargino mass. The horizontal line is the present experimental lower bound on the chargino mass.*

and, in SO(10) models, also by the $D$-term of the broken SO(10)/SU(5) = U(1)$_X$ factor.

The parameters at the electroweak scale are obtained as [9]

$$m^2_{h^{\rm u}} = m^2_{h^{\rm u}}(M_{\rm GUT}) + 0.79 M_2^2 - \frac{1}{2} 3I \quad (4a)$$

$$m^2_{\tilde{Q}_3} = m^2_{\tilde{Q}_3}(M_{\rm GUT}) + 10.6 M_2^2 - \frac{1}{6} 3I \quad (4b)$$

$$m^2_{\tilde{t}_R} = m^2_{\tilde{t}_R}(M_{\rm GUT}) + 10.0 M_2^2 - \frac{1}{3} 3I \quad (4c)$$

with

$$3I = \rho[X_t(M_{\rm GUT}) + 3\rho(6.5 - 2.5\rho)M_2^2] + \\ +\rho(1-\rho)[A_{t\rm G}^2 + 5.5 M_2 A_{t\rm G}],$$

where $X_t \equiv m^2_{\tilde{Q}_3} + m^2_{\tilde{t}_R} + m^2_{h^{\rm u}}$, $A_{t\rm G}$ is the trilinear term for the top Yukawa coupling at the unification scale, and $\rho = (\lambda_t(M_Z)/\lambda_{tZ}^{\rm max})^2$, where $\lambda^{\rm max}_{tZ} \approx 1.14$ is the maximum value that $\lambda_t(M_Z)$ can reach without developing a Landau pole below the unification scale. Note that $\rho \approx 1$ near the infrared fixed point for $\lambda_t$, so that the dependence of the scalar masses on $A_{t\rm G}$ is negligible. Note also that the gaugino contribution to the soft masses does not depend on the unknown boundary condition for the scalar masses. This shows that the fine-tuning constraints on $M_2$ are not model-dependent.

Requiring that the $\tilde{t}_R$ and $h^{\rm u}$ soft mass$^2$ terms be negative give rise to the restrictions on the parameter space shown in figure 1. Let us explain its features in a qualitative way. Above the upper lines (at big $x$) $m^2_{\tilde{t}_R}$ is so big that cannot be driven to be negative by the $\lambda_t$ effect in the renormalization group equations (RGEs). Below the lower lines (at small $x$) the same happens for $m^2_{h^{\rm u}}$, preventing the breaking of the electroweak gauge group. As shown in fig. 1, for moderate values of $y$ the two bounds are compatible only in a restricted range of $x$, $0.5 \lesssim x \lesssim 0.9$, and for small values of $M_2 \lesssim m_{10}/4$, where $M_2$ is renormalized at the electroweak scale. This particular restriction on the parameter space also suggests the presence of a light chargino. Note also that with universal soft terms it is not possible to get a sufficiently light right-handed stop, at least unless $A_{t\rm G}$ is very large.

We now plot, in figure 2, the values of the lightest chargino mass as function of the required fine-tuning, as defined previously, for arbitrary random possible values of the various parameters $\lambda_t$, $\tan\beta$, $\mu$, $m_{10}$, $M_2$, $A_{t\rm G}$, $x$ and with $y$ in the range $0.5 \div 1.5$. We see that a lightest chargino not very light, $M_\chi \gtrsim M_Z$, is possible only with a considerable amount of fine tuning, beyond the % level and that it is impossible to require a fine tuning weaker than the 10% level, as done in [5], without contradicting the experimental lower bound on the chargino mass. Since the lightest chargino is very light, finite one loop quantum corrections that we have not included can increase its mass by $(3 \div 10)\%$ [10].

We now discuss the stop sector, finding a nice consistency of the various bounds. The solution of the RGE for $A_t$ is [9]

$$A_t = (1-\rho)A_{t\rm G} + (4.9 - 2.8\rho)M_2$$

so that $A_t \sim (1 \div 3.5)M_2$ near the infrared fixed point for the top Yukawa coupling[‡]. This is a welcome restriction. In fact $A_t$ cannot be too large, both because the condition

$$A_t^2 + 3\mu^2 \lesssim 7.5(m^2_{\tilde{t}_R} + m^2_{\tilde{t}_L})$$

must be satisfied to avoid a too fast decay of the SM-like minimum into charge and color breaking minima [4], and both in order that the lightest eigenstate of the stop mass matrix

$$\begin{pmatrix} m^2_{\tilde{Q}_3} + m_t^2 + 0.35 M_Z^2 & -m_t(A_t^* + \mu\cot\beta) \\ -m_t(A_t + \mu^*\cot\beta) & m^2_{\tilde{t}_R} + m_t^2 + 0.15 M_Z^2 \end{pmatrix}$$

be almost right-handed (for $m^2_{\tilde{t}_R} \lesssim 0 < m^2_{\tilde{Q}_3}$). We now plot in figure 3, again for random samples of the free parameters and near the infrared fixed point of the top quark Yukawa coupling, the possible values of

[‡]The stronger forms for the $M_2/A_t$ and $M_2/\mu$ correlations that appear in the literature [11] hold for large values of $M_2$ that, in our case, are disfavored by naturalness bounds.

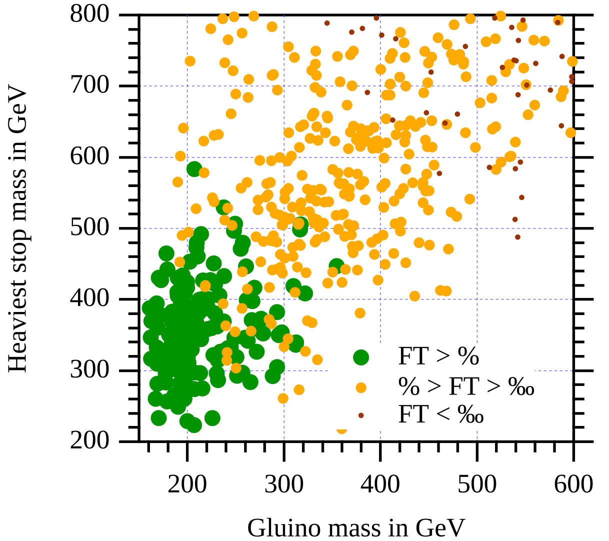


Figure 3: *Values of* $(M_3, M_{\tilde{T}})$ *for weak, strong or very strong fine-tuning* FT.

the gluino mass and of the heaviest stop mass, distinguishing the cases of moderate fine tuning, $\Delta < 100$, (big green dots), strong fine tuning, $100 < \Delta < 1000$, (medium orange dots) and very strong fine tuning, $\Delta > 1000$, (small red dots). We have imposed the experimental lower bounds on the masses of the supersymmetric particles, and in particular the bound on the chargino mass, $M_\chi > 65\,\text{GeV}$. Since $A_t$ is small and positive (due to its correlation with $M_2$), while $\mu$ has most likely the opposite sign (see fig. 2), and for the values of the heaviest stop mass suggested in figure 3, the left/right mixing angle $\theta_{\tilde{t}}$ in the stop sector is small, $|\theta_{\tilde{t}}| \lesssim 0.3$. It is interesting that this is the appropriate amount of mixing necessary to get a sufficiently light and right-handed stop state, $m_{\tilde{t}} \lesssim 100\,\text{GeV}$, without encountering problems with dangerous unphysical minima [1, 4] or with undesired radiative corrections [2]. In conclusion, we summarize the naturalness bounds on $M_3$ and $M_{\tilde{T}}$ as

$$M_3 \lesssim 300\,\text{GeV}\sqrt[4]{\frac{\Delta_{\text{lim}}}{100}}, \qquad M_{\tilde{T}} \lesssim 500\,\text{GeV}\sqrt[4]{\frac{\Delta_{\text{lim}}}{100}}.$$

Finally, the dependence of $M_Z^2$ and $m_{\tilde{t}_R}^2$ on the masses of the sfermions of first and second generation only comes, at one loop, through a small term, neglected in eq. (4), proportional to $X_Y \equiv \sum N_R Y_R m_R^2$, where the sum runs over all the MSSM scalar fields $R$ with $N_R$ components and hypercharge $Y_R$. The associated fine-tuning upper bound, $X_Y \lesssim \text{TeV}(10/\Delta_{\text{lim}})^{1/2}$, has been computed in ref. [12]. Adding the requirement of a light stop it becomes $X_Y \lesssim \text{TeV}(100/\Delta_{\text{lim}})^{1/4}$.

**4** It is also possible that, differently from what we have assumed so far, the soft terms appear as point-like terms only up to some energy $M_U$ below the unification scale. For example supersymmetry breaking could be directly felt by appropriate gauge charged 'transmitter' fields with mass $\sim M_U$ below or around the gauge-unification scale [7]. If this is the case unification physics would not be reflected in the soft terms, and minimal models of this kind agree on the predictions

$$M_i \propto \alpha_i, \qquad m_R^2 \propto c_i^R \alpha_i^2 \qquad \text{at } M_U \tag{5}$$

where $c_i^R$ are the traces, in the representation $R$, of the generators of the three factors $G_i$ of the SM gauge group. In particular we have $m_{\tilde{Q}_3}^2 \sim m_{\tilde{t}_R}^2 \sim 2m_{h^u}^2$ at $M_U$, so that we can see from fig. 1 that it seems not possible to get a light right-handed stop in this case, neither if $M_U$ were not much smaller than $M_{\text{GUT}}$. We must however remind that in such models it is difficult to understand the origin of the supersymmetric $\mu$-term, so that some important amount of non-minimality (for example in the messenger field content [13] or in the light fields [14]) could play a decisive role.

# References


[1] M. Carena, M. Quirós and C.E.M. Wagner, *Phys. Lett.* **B380** (1996) 81; J.R. Espinosa, hep-ph/9604320; D. Delepine, J.-M. Gerard, R. Gonzalez Felipe and J. Weyers, hep-ph/9604440.

[2] for a recent analysis see W. de Boer et al., hep-ph/9609209, and references therein.

[3] see for example M. Shifman, *Int. J. Mod. Phys.* **A11** (1996) 3195.

[4] A. Kusenko, P. Langacker and G. Segre, hep-ph/9602414.

[5] R. Barbieri and G.F. Giudice, *Nucl. Phys.* **B306** (1988) 63.

[6] R. Barbieri, S. Ferrara and C.A. Savoy, *Phys. Lett.* **119B** (1982) 343; P. Nath, R. Arnowitt and A. Chamseddine, *Phys. Rev. Lett.* **49** (1982) 970.

[7] L. Alvarez-Gaume, M. Claudson and M.B. Wise, *Nucl. Phys.* **B207** (1982) 96.

[8] L.J. Hall, J. Lykken and S. Weinberg, *Phys. Rev.* **D10** (1983) 2359; G.F. Giudice and E. Roulet, *Phys. Lett.* **B315** (1993) 107.

[9] L. Ibáñez, C. Lopez and C. Muñoz, *Nucl. Phys.* **B256** (1985) 218; A. Boquet, J. Kaplan and C.A. Savoy, *Nucl. Phys.* **B262** (1985) 299; R. Barbieri, L. Hall and A. Strumia, *Nucl. Phys.* **B445** (1995) 219.

[10] D.M. Pierce, J.A. Bagger, K. Matchev and R. Zhang, hep-ph/9606211.

[11] M. Carena, M. Olechowski, S. Pokorski and C.E.M. Wagner, *Nucl. Phys.* **B419** (1994) 213 and *Nucl. Phys.* **B426** (1994) 269; M. Carena and C.E.M. Wagner, *Nucl. Phys.* **B452** (1995) 45.

[12] S. Dimopoulos and G.F. Giudice, *Phys. Lett.* **B357** (1995) 573.

[13] G. Dvali, G.F. Giudice and A. Pomarol, hep-ph/9603238.

[14] H. P. Nilles, M. Srednicki and D. Wyler, *Phys. Lett.* **B120** (1983) 346; M. Dine and A. E. Nelson, *Phys. Rev.* **D48** (1993) 1277.